10101 **A**

Reg. No.:__________________ Name: ____________________

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JANUARY 2016

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each question carries 3 marks

1. Show that the series $\sum_{k=1}^{\infty} \frac{\cos k}{k^2}$ is convergent.
2. Find $\frac{d}{dx}\left(e^x \operatorname{sech}^{-1}\sqrt{x}\right)$
3. Identify the surfaces $5x^2 - 4y^2 + 20z^2 = 0$
4. Equation of a surface in spherical coordinates is $\rho = \sin\theta \sin\varphi$
   Find the equation of this surface in rectangular coordinates.
5. Given $f = e^x \sin y$; show that the function satisfies the Laplace equation $f_{xx} + f_{yy} = 0$
6. Let $w = 4x^2 + 4y^2 + z^2$;where $x = \rho \sin\varphi \cos\theta,\ y = \rho \sin\varphi \sin\theta,\ z = \rho \cos\varphi$ Find $\frac{\partial w}{\partial \rho}$ using chain rule.
7. A particle moves along a circular helix in 3-space so that its position vector at time t is $r(t) = (4\cos \pi t)i + (4\sin \pi t)j + tk$ Find the displacement of the particle during theinterval $1 \le t \le 5$.
8. Find the tangent to the curve $r(t) = (t^2 - 1)\vec{\imath} + t\vec{\jmath}$ at $t = 1$
9. Evaluate $\int_1^a \int_1^b \frac{dydx}{xy}$
10. The line y = 2- x and the parabola $y = x^2$ intersect at the points (-2, 4) and (1, 1).If R is the region enclosed by y=2-x and $y=x^2$,then find $\iint_R (y)\, dA$

(10 x 3 = 30 Marks)

PART B

Answer any 2 complete questions each having 7 marks

11. Find the radius of convergence and interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$.
12. Test the convergence of $\frac{x}{12} + \frac{x^2}{23} + \frac{x^3}{34} + \cdots \ldots \ldots$
13. Find the Taylors series of $\frac{1}{x}$ about x = 1.

Answer any 2 complete questions each having 7 marks

14. Find the domains of (i) $f(x,y)=\sqrt{25-x^2-y^2-z^2}$ (ii) $f(x,y)=\ln(x-y^2)$ and describe them in words.
15. Find the limit of $f(x,y) = \frac{-xy}{x^2+y^2}$ as (x,y)→ (0,0) along (i) the X-axis , (ii) the Y- axis (iii) the line y = x.
16. Find the spherical and cylindrical coordinates of the point that has rectangular coordinates(x,y,z)=(4,-4,4$\sqrt{6}$)

Answer any 2 complete questions each having 7 marks

17. Locate all relative maxima, relative minima and saddle point if any,of $f(x,y) = y^2 + xy + 4y + 2x + 3$
18. Let f be a differentiable fuction of 3 varaiables and suppose that $W = f(x-y, y-z, z-x)$. Prove that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$.
19. Find the local linear approximation L(x,y) to f(x,y) $= \frac{1}{\sqrt{x^2+y^2}}$ at the point P(4,3). Compare the error in approximating 'f' by L at the specified point Q (3.92, 3.01) with the distance between P and Q.

Answer any 2 complete questions each having 7 marks

20. Find $y(t)$ where $y''(t) = 12t^2 i - 2tj$, $y(0) = 2i - 4j$, $y'(0) = 0$.
21. Find the arc length parametrization of the line x = 1 + t,y = 3 − 2t, z = 4 + 2t that has the same direction as the given line and has reference point (1, 3, 4).
22. Find the directional derivative of f(x, y) = e^xsec y at P (0, π/4) in the direction of PQ where Q is the origin.

Answer any 2 complete questions each having 7 marks

23. Find the area bounded by the x-axis, $y = 2x$ and x+y=1 using double integration.
24. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.
25. Sketch the region of integration and evaluate the integral $\int_1^2 \int_y^{y^2} dxdy$ by changing the order of integration.