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10251

Reg. No.________________ Name: ____________________________

SECOND SEMESTER B.TECH DEGREE EXAMINATION, JULY 2016

Course Code: MA-102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 hrs

PART A

Answer all questions Each carries 3 marks

(1) Find the general solution of $y''' - y = 0$

(2) Find the wronskian of the following $e^{-x}\cos 5x$; $e^{-x}\sin 5x$

(3) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$

(4) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2$

(5) Express $f(x) = x$ as aFourier series in the interval $-\pi < x < \pi$

(6) Obtain the half range Fourier sine series for the function e^x in $0 < x < 2$

(7) Form the partial differential equation by eliminating the arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

(8) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$

(9) Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 3e^{-5x}$

(10) State the one dimensional wave equation with boundary conditions and initial conditions for solving it

(11) In the Heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ what does α^2 indicate. State the boundary and initial conditions for solving it

(12) Find the steady state temperature distribution in a rod of length 25cm, if the ends of the rod are kept at 20^0c and 70^0c.

PART B

Answer one full question from each module

Module -I

(13) (a) Solve $y''' - 8y'' + 37y' - 50y = 0$ (6)

(b)Determine all possible solutions to the initial value problem $y' = 1 + y^2, y(0) = 0 \;\; in \;\; |x| < 3, |y| < 2$ (5)

OR

(14) (a)Find the general solution of $y^{iv} - y''' - 9\ y'' - 11\ y' - 4\ y = 0$ (6)

(b)Determine all possible solutions to the initial value problem

$y' = y^{\frac{1}{2}}, y(0) = 0.$ (5)

Module - II

(15) (a)Solve by method of variation of parameters $\frac{d^2y}{dx^2} + y = xsinx.$ (6)

(b)Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x.$ (5)

OR

(16) (a)Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x.$ (6)

(b)Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = sin3xsin2x$. (5)

Module - III

(17) (a)Obtain the Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x \leq \pi \end{cases}$$ (6)

(b)Obtain the Fourier series to represent the function

$f(x) = |sinx|; -\pi < x < \pi$ (5)

OR

(18) (a)Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \leq x \leq$ (6)

(b)Find the half range cosine series for the function $f(x) = x^2$ in the range $0 \leq x \leq \pi$ (5)

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Module - IV

(19) (a)Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 5e^{3x} - 7x^2 y.$ (6)

(b)Solve $(x+y)zp + (x-y)zq = x^2 + y^2$ (5)

OR

(20) (a) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x+2y).$ (6)

(b)Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = cos2x\ cos3y.$ (5)

Module – V

(21) A tightly stretched string with fixed end points $\boldsymbol{x = 0\ and\ x = l}$ is initially in a position given by $y = y_0 sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $\boldsymbol{y(x,t)}$. (10)

OR

(22) A tightly stretched string with fixed end points $\boldsymbol{x = 0\ and\ x = l}$ *is* initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $\boldsymbol{\lambda x(l-x)}$, find the displacement of the string at any distance x from one end at any time $\boldsymbol{t}$. (10)

Module - VI

(23) A bar 10 cm long with insulated sides has its ends A and B maintained at 30^0c and 100^0 c respectively until steady state conditions prevail. The temperature at A is suddenly raised to 20^0 c and at the same time that of B is lowered to 40^0 C. Find the temperature distribution in the bar at time t. (10)

OR

(24) A rod of 30cm long has its ends A and B kept at 30^0 c and 90^0 c respectively until steady state temperature prevails. The temperature at each end is then suddenly reduced to zero temperature and kept so. Find the resulting temperature function $\boldsymbol{u(x,y)}$ taking $x = 0$ at A. (10)